Timing and Time Inconsistency in Search Models*

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Abstract

We study how timing assumption matters for the commitment problem and time inconsistency in search models with unemployment insurance. We analyze the Markov equilibrium without commitment and the Ramsey policy with commitment under two different timings. The first timing is where consumption takes place before search within each period, and the second timing is where search takes place before consumption. We show that time inconsistency occurs mainly through search disincentive under the first timing, but only indirectly under the second timing. We show theoretically and numerically that the magnitude of time inconsistency is stronger under the first timing.

Keywords: Time inconsistency issue, Markov-perfect equilibrium, Ramsey policy, Unemployment insurance, Search model
JEL classifications: E61, J64, J65, H21

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1 Introduction

A large literature studies the moral hazard problem of unemployment insurance (UI) in discouraging job creations in the labor market. Optimal UI policy design in the search-and-matching framework generally focuses on the trade-off between providing insurance to unemployed workers and creating incentives to search. However, a time inconsistency issue arises: ex ante the government wants to promise less generous UI to encourage job creation; ex post it wants to give more generous UI for income insurance. This time inconsistency means that the government's commitment to future policies matters. Compared to other model contexts, the time inconsistency issue in search model is less well understood (see, for example, Pei and Xie 2021, and Mitman and Rabinovich 2021).

In this paper, we show how timing assumption matters for the extent of time inconsistency in a search model. We start with two alternative timing assumptions to illustrate the difference. Under the first timing, consumption takes place before search within each period; under the second timing, search takes place before consumption. Under both timings, we characterize the UI policy with discretion (i.e., without commitment) and the policy with commitment. Time inconsistency arises under the first timing: before unemployed workers search, the government wants to promise lower future UI benefits to encourage search, but after search has taken place the government wants to give higher UI benefits. In contrast, under the second timing, current-period benefit does not have a direct effect on search in the previous period, although it indirectly affects the previous-period search via the current-period search. Thus, the time inconsistency issue, though still exists, takes a much more indirect form. We show theoretically and numerically that time inconsistency is weaker under the second timing, as the government with discretion under this timing is able to internalize the disincentive effects of its policy on current-period search. We also show that the timing assumption matters for the UI policy with discretion, but mostly does not matter for the policy with commitment.

This paper contributes to the literature on optimal UI.[1] The importance of timing highlighted here contributes to the understanding of time inconsistency and policies with and without commitment in search models. While the focus here is on the search model, the reasoning can also be applied to time inconsistency issues in the capital taxation literature and the inflation literature. In a similar spirit, Ortigueira (2006) demonstrates that timing assumption is important for the time-consistent policy (with discretion) in optimal taxation, while Chugh (2009) shows that the Ramsey optimal policy (with commitment) is not sensitive to timing assumption in an economy with cash-in-advance constraint.

2 A Simple Search Model

The model is based on a Diamond-Mortensen-Pissarides framework. Time is discrete and infinite, and there are no aggregate shocks. The model economy consists of a measure one of infinitely-lived workers. In any given period, a worker can be either employed or unemployed. Workers are risk

[1] See, for example, Hopenhayn and Nicolini (1997), Wang and Williamson (2002), Young (2004), Shimer and Werning (2008), Chetty (2008), Mitman and Rabinovich (2015), Jung and Kuester (2015), and Landais et al. (2018).

averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - v(s_t) \right],$$

where $\mathbb{E}_0$ is the period-0 expectation operator, and β is the time discount factor. The period utility $U(c) - v(s)$ comprises of utility from consumption c and disutility from job search s. Utility is increasing and concave in c, and decreasing and convex in s.

Each period, an employed worker is paid wage w from production. Unemployed workers search for jobs, and we assume employed workers do not search (i.e., no on-the-job search). An unemployed worker receives UI benefits b, and also derives utility from nonmarket activity h, which we take as the combined value of leisure, home production and welfare. There are no private insurance markets and workers cannot save or borrow. Workers consume their income—w for the employed, and $h + b$ for the unemployed—net of taxes.

To focus on the disincentives of UI on search, we abstract from firms in the model, although the results in the paper continue to hold with firms and aggregate shocks. The job-finding probability per efficiency unit of search is f, and thus the job finding probability for an unemployed searching with intensity s is fs. Employed workers are separated from their jobs each period with the job separation probability δ.

The government decides on the generosity of the UI program by varying the benefit level b. The government finances UI through a lump-sum tax, τ, on all workers, both employed and unemployed, and needs to balance its budget every period.

3 The First Timing

Timing of actions matters in this simple search model, and we illustrate by comparing two alternative timings. Under the first timing (Timing 1), consumption and production take place *before* search within each period, as illustrated in Figure 1. The economy enters a period t with a level of unemployment u_t. Once government policies (b_t, τ_t) for the period are decided, unemployed workers collect benefits b_t, and employed workers receive wage w. Everyone pays a lump-sum tax τ_t and consumes after-tax income. Unemployed workers then choose search intensity s_t, and the fraction of unemployed workers who find jobs is fs_t. At the same time, a fraction δ of employed workers are separated. The unemployment level at the end of the period (or the beginning of the next period), u_{t+1}, is determined according to the following law of motion,

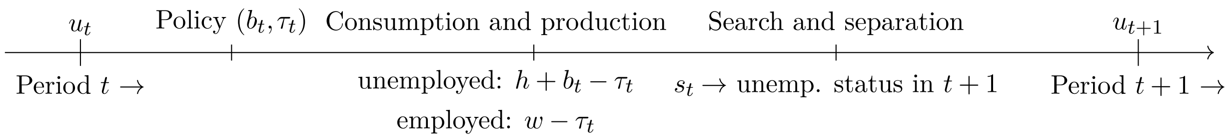
$$u_{t+1} = \delta(1 - u_t) + (1 - fs_t)u_t. \tag{1}$$

The period-t government budget constraint is

$$\tau_t = u_t b_t. \tag{2}$$

Under this timing, because consumption (and production) take place before search within a period, search in period-t affects the employment status and thus consumption in period-$t+1$. Therefore, current benefit b_t does not have a direct effect on search s_t today. It is the expected

Figure 1: Timing 1 of Search-Consumption Problem



future benefit b_{t+1} that affects s_t today. We can see this more clearly from the unemployed worker's problem.

3.1 Worker's problem

Let $V^e(u_t)$ and $V^u(u_t)$ be the values of an employed and an unemployed worker, respectively, with the beginning-of-period unemployment u_t. The Bellman equation of an employed worker is then

$$V^e(u_t) = U(w - \tau_t) + (1-\delta)\beta V^e(u_{t+1}) + \delta\beta V^u(u_{t+1}).$$

An unemployed worker's optimization problem is

$$V^u(u_t) = \max_{s_t} \quad U(h + b_t - \tau_t) - v(s_t) + f s_t \beta V^e(u_{t+1}) + (1 - f s_t)\beta V^u(u_{t+1}).$$

The optimal choice of search intensity s_t for the unemployed worker is characterized by

$$\frac{v'(s_t)}{f} \quad = \quad \beta\left[U(w - \tau_{t+1}) - U(h + b_{t+1} - \tau_{t+1}) + v(s_{t+1}) + (1 - \delta - f s_{t+1})\frac{v'(s_{t+1})}{f}\right]. \qquad (3)$$

The unemployed worker's optimality condition equates the marginal cost (left-hand side) of increasing search intensity to the marginal benefit (right-hand side). The marginal cost is the marginal disutility of search weighted by the job finding efficiency rate. The marginal benefit is the sum of utility gain from being employed next period and the benefit of economizing on future search cost. A higher future benefit b_{t+1} lowers the utility gain from being employed next period, and thus reduces search incentive today.

Time inconsistency. Equation (3) helps highlight the main source of time inconsistency. Note that b_{t+1} is chosen by the government at the start of period $t+1$, after s_t has been carried out by the unemployed worker. As such, before search takes place, the government can promise less generous future UI benefits to encourage job search, but after search has been determined, the government can implement more generous policies for insurance purposes.

3.2 UI policy with discretion

Because of the time inconsistency, the government's commitment to future policies matters. To illustrate how (much) time inconsistency matters, it helps to compare the government's problem with commitment and with discretion (without commitment). The larger the time inconsistency issue, the more the policy with discretion deviates from the policy with commitment.

We assume that the government is a utilitarian planner, who maximizes the expected value of the worker's utility using policy UI benefit b and tax τ. We first describe the government's problem with discretion, by characterizing a Markov equilibrium that depends differentiably on the payoff-relevant aggregate state – u in this case, following Klein et al. (2008). The period-welfare function is equal to the average utility of all workers, given by

$$R(u_t, b_t, s_t) = (1-u_t)U(w-\tau_t) + u_t\left[U(h+b_t-\tau_t) - v(s_t)\right].$$

Definition 1. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of government's value function G, government policy rule Ψ, and private decision rules $\{S, \Gamma\}$ such that for any aggregate state u_t, the policies $b_t = \Psi(u_t)$, $s_t = S(u_t)$ and $u_{t+1} = \Gamma(u_t)$ solve

$$\max_{b_t, s_t, u_{t+1}} \qquad R(u_t, b_t, s_t) + \beta G(u_{t+1}),$$

subject to the flow equation (1), the optimal search condition (3), the government's budget constraint (2), and the government value function satisfies the functional equation

$$G(u_t) \quad \equiv \quad R(u_t, \Psi(u_t), S(u_t)) + \beta G(\Gamma(u_t)). \tag{4}$$

Note that the worker's optimal search condition (3) is also the incentive constraint in the government's problem. The no-commitment assumption on the government is reflected in the Markovian setup, where the policy functions are time independent and are only functions of the current aggregate state. Equation (4) reflects that future governments follow the same policy rules $\{\Psi, S, \Gamma\}$, thereby making the policies time-consistent.

For ease of exposition, we denote the flow equation (1) by $\eta_0(u_t, s_t, u_{t+1}) = 0$ and the optimal search condition (3) by $\eta_1(s_t, u_{t+1}; \Psi, S) = 0$. Let λ_t and μ_t be the Lagrange multipliers on η_0 and η_1 respectively. The Markov equilibrium can be characterized by the flow equation (1), the worker's optimal search condition (3), and the following government's first-order conditions with respect to b_t, s_t and u_{t+1}:

$$b_t: \qquad \frac{\partial R_t}{\partial b_t} = 0 \tag{5}$$

$$s_t: \qquad \lambda_t \frac{\partial \eta_{0,t}}{\partial s_t} + \mu_t \frac{\partial \eta_{1,t}}{\partial s_t} - \frac{\partial R_t}{\partial s_t} = 0 \tag{6}$$

$$u_{t+1}: \qquad \lambda_t \frac{\partial \eta_{0,t}}{\partial u_{t+1}} + \mu_t \frac{\partial \eta_{1,t}}{\partial u_{t+1}} - \beta\left(\frac{\partial R_{t+1}}{\partial u_{t+1}} - \lambda_{t+1}\frac{\partial \eta_{0,t+1}}{\partial u_{t+1}}\right) = 0 \tag{7}$$

Optimal benefit equation (5) summarizes the welfare effects of changing UI benefits: $\frac{\partial R_t}{\partial b_t} \equiv u_t(1-u_t)U^u_{c,t} - (1-u_t)u_t U^e_{c,t} = 0$. The equation shows that when the government chooses UI benefit, it considers only the marginal effect of redistribution between the employed and the unemployed workers. The government does not consider the incentive effect of a higher b_t on unemployed workers' search. This is because, given the timing, the government with discretion cannot promise a low b_t today to incentivize search in the previous period. As such, the government is unable to use b_t as an incentive device, and only uses it to equalize the weighted marginal utilities of consumption between the employed and the unemployed workers in the current period.

3.3 UI policy with commitment

In contrast, the government with commitment can set benefit to create incentives for search. We characterize the problem of the Ramsey government, who makes contingent policies for all future periods at time zero and is assumed to follow these pre-determined plans.

Definition 2. (Ramsey problem) Given an initial unemployment u_0, the Ramsey government policy consists of a sequence of benefits $\{b_t\}_{t=0}^{\infty}$ and private allocations $\{s_t, u_{t+1}\}_{t=0}^{\infty}$ that solves

$$\max_{\{b_t, s_t, u_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t R(u_t, b_t, s_t),$$

subject to the flow equation (1), the optimal search condition (3), and the government's budget constraint (2) for all time t.

Similar to before, we use $\tilde{\eta}_0(u_t, s_t, u_{t+1}) = 0$ and $\tilde{\eta}_1(s_t, u_{t+1}, b_{t+1}, s_{t+1}) = 0$ to denote the flow equation (1) and the optimal search condition (3) respectively. Let $\beta^t \tilde{\lambda}_t$ and $\beta^t \tilde{\mu}_t$ be the Lagrange multipliers on $\tilde{\eta}_0$ and $\tilde{\eta}_1$ respectively. The optimal Ramsey government policy can be characterized by the flow equation (1), the worker's optimal search condition (3), and the following government's first-order conditions with respect to b_t, s_t and u_{t+1} for all $t > 0$,

$$b_t: \qquad \frac{\tilde{\mu}_{t-1}}{\beta} \frac{\partial \tilde{\eta}_{1,t-1}}{\partial b_t} - \frac{\partial R_t}{\partial b_t} = 0 \tag{8}$$

$$s_t: \qquad \frac{\tilde{\mu}_{t-1}}{\beta} \frac{\partial \tilde{\eta}_{1,t-1}}{\partial s_t} + \tilde{\lambda}_t \frac{\partial \tilde{\eta}_{0,t}}{\partial s_t} + \tilde{\mu}_t \frac{\partial \tilde{\eta}_{1,t}}{\partial s_t} - \frac{\partial R_t}{\partial s_t} = 0 \tag{9}$$

$$u_{t+1}: \qquad \tilde{\lambda}_t \frac{\partial \tilde{\eta}_{0,t}}{\partial u_{t+1}} + \tilde{\mu}_t \frac{\partial \tilde{\eta}_{1,t}}{\partial u_{t+1}} - \beta \left(\frac{\partial R_{t+1}}{\partial u_{t+1}} - \tilde{\lambda}_{t+1} \frac{\partial \tilde{\eta}_{0,t+1}}{\partial u_{t+1}} \right) = 0 \tag{10}$$

Compared to the optimal benefit condition for the government with discretion (equation 5), the Ramsey government's optimal benefit condition (8) has an extra term (in red highlight). This term contains two parts: $\partial \tilde{\eta}_{1,t-1} / \partial b_t$ is the effect of a higher benefit b_t on the unemployed worker's private marginal value of search in period $t-1$; the lagged Lagrange multiplier $\tilde{\mu}_{t-1}$ is the social marginal value of relaxing the period-$t-1$ optimal search condition of unemployed workers. Together, the extra term captures the effect of period-t benefit b_t on period-$t-1$ search by unemployed workers. The presence of the term in the optimal benefit condition means the period-t Ramsey policy takes into account incentives of unemployed workers in period $t-1$. In other words, the policy in period t has to be consistent with the promised marginal values for period $t-1$, which reflects the government's commitment in the Ramsey problem. The extra term in equation (9) similarly reflects this commitment. Because the government with commitment takes into account the disincentive effect of a higher b_t, we have the following proposition:

Proposition 1. The benefit level with discretion is *higher* than the benefit level with commitment.

Proof. The government with discretion chooses benefits according to equation (5), which is to set b_t to equalize marginal utilities of consumption between the unemployed and the employed workers: $U^u_{c,t} = U^e_{c,t}$. The government with commitment chooses benefits according to equation (8), which simplifies to $\mu_{t-1}\left[u_t U^e_{c,t}+(1-u_t)U^u_{c,t}\right]-\frac{\partial R_t}{\partial b_t}=0$. As the marginal utilities and the multiplier are positive, the first term is positive. Thus $\frac{\partial R_t}{\partial b_t}>0$, which implies $U^u_{c,t}>U^e_{c,t}$. This means the benefit level with commitment is not high enough to equalize marginal utilities, and thus it is lower than the benefit with discretion. $\square$

4 The Second Timing

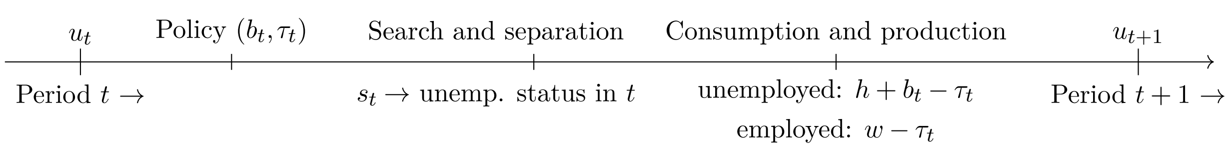
Now consider an alternative timing (Timing 2). For convenience, we continue to use the same notations as under Timing 1, although the equilibrium values can be different in these two different setups. Under this timing, consumption and production take place *after* search within each period, as illustrated in Figure 2. Given the unemployment u_t at the beginning of period t, the government decides this period's policies (b_t, τ_t). Then job search s_t and job separation take place. After that, consumption and production are realized. The workers who were not separated or who found a job this period receive wage w. The workers who were newly separated or who did not find a job receive benefits b_t. The unemployment flow equation is the same as under Timing 1:

$$u_{t+1}=\delta(1-u_t)+(1-fs_t)u_t. \tag{11}$$

The period-t government budget constraint is different from Timing 1:

$$\tau_t=\left[\delta(1-u_t)+(1-fs_t)u_t\right]b_t=u_{t+1}b_t. \tag{12}$$

Figure 2: Timing 2 of Search-Consumption Problem



Note that because search and separation take place before consumption within each period, search in period t determines the employment status and consumption in the same period, and UI benefits are given to workers who are unemployed at the end of the period. As such, current-period benefit b_t directly affects search s_t in the same period.

4.1 Worker's problem

The Bellman equation of an employed worker is

$$V^e(u_t)=(1-\delta)\left[U(w-\tau_t)+\beta V^e(u_{t+1})\right]+\delta\left[U(h+b_t-\tau_t)+\beta V^u(u_{t+1})\right].$$

An unemployed worker's optimization problem is

$$V^u(u_t)=\max_{s_t}\quad -v(s_t)+fs_t\left[U(w-\tau_t)+\beta V^e(u_{t+1})\right]+(1-fs_t)\left[U(h+b_t-\tau_t)+\beta V^u(u_{t+1})\right].$$

The optimal choice of search intensity s_t for the unemployed worker is characterized by

$$\frac{v'(s_t)}{f}\quad=\quad U(w-\tau_t)-U(h+b_t-\tau_t)+\beta\left[v(s_{t+1})+(1-\delta-fs_{t+1})\frac{v'(s_{t+1})}{f}\right]. \tag{13}$$

Similar to before, the unemployed worker's optimal search condition (13) equates the marginal cost of higher search (left-hand side) to the marginal benefit (right-hand side). But different from the condition under Timing 1, here the current-period benefit b_t (instead of the next-period benefit b_{t+1}) appears as part of the marginal benefit from search and thus directly affects the unemployed worker's search decision today. A higher benefit b_t today reduces the utility gain from employment and lowers the marginal value of search today.

Time inconsistency. Under this timing, benefit b_t is set by the government before search s_t is chosen, unlike under Timing 1 where b_{t+1}, which directly affects s_t, is chosen after s_t is decided. Thus, the government under this timing cannot promise a low benefit to directly incentivize search — because b_t has already been decided when s_t takes place. In other words, there is no direct time inconsistency in b_t under this timing. However, there is an indirect time inconsistency.[2] Period-$t+1$ search s_{t+1} appears on the right-hand side of the worker's optimal search condition, as part of the marginal benefit to more search in period t: higher search in period t increases the chance of employment and saves on search for period $t+1$. Because period-$t+1$ benefit affects period-$t+1$ search, it then indirectly affects period-t search in equilibrium. Therefore, the government can promise a lower b_{t+1}, which raises the unemployed worker's search s_{t+1} in period $t+1$, and in turn raises current-period search s_t; but once workers have chosen s_t, the government can set a higher b_{t+1} to redistribute income from the employed to the unemployed. This constitutes the indirect time inconsistency under this timing.

4.2 UI policy with discretion

Under Timing 2, the period-welfare function is

$$R(u_t, b_t, s_t) = [(1-\delta)(1-u_t) + f s_t u_t]\, U(w-\tau_t) + [\delta(1-u_t) + (1-f s_t)u_t]\, U(h+b_t-\tau_t) - u_t v(s_t).$$

The Markov equilibrium is similarly defined as that under Timing 1. Denote the flow equation (11) by $\eta_0(u_t, s_t, u_{t+1}) = 0$ and the optimal search condition (13) by $\eta_1(u_t, b_t, s_t, u_{t+1}; S) = 0$. Let λ_t and μ_t be the Lagrange multipliers on η_0 and η_1 respectively. The Markov equilibrium under Timing 2 can be characterized by the flow equation (11), the worker's optimal search condition (13), and the following government's first-order conditions:

$$b_t: \qquad \mu_t \frac{\partial \eta_{1,t}}{\partial b_t} - \frac{\partial R_t}{\partial b_t} = 0 \tag{14}$$

$$s_t: \qquad \lambda_t \frac{\partial \eta_{0,t}}{\partial s_t} + \mu_t \frac{\partial \eta_{1,t}}{\partial s_t} - \frac{\partial R_t}{\partial s_t} = 0 \tag{15}$$

$$u_{t+1}: \qquad \lambda_t \frac{\partial \eta_{0,t}}{\partial u_{t+1}} + \mu_t \frac{\partial \eta_{1,t}}{\partial u_{t+1}} - \beta \left(\frac{\partial R_{t+1}}{\partial u_{t+1}} - \lambda_{t+1} \frac{\partial \eta_{0,t+1}}{\partial u_{t+1}} - \mu_{t+1} \frac{\partial \eta_{1,t+1}}{\partial u_{t+1}} \right) = 0 \tag{16}$$

Under this timing, because search is directly affected by current-period benefit policy, the government with discretion considers not only the redistributional effect of b_t between the employed and the unemployed workers ($\partial R_t / \partial b_t$), but also the incentive effect of b_t on unemployed workers' search this period ($\partial \eta_{1,t} / \partial b_t$). As such, the government faces the trade-off between redistribution

[2] This indirect time inconsistency is also present under Timing 1, together with the main source of time inconsistency.

and minimizing the disincentive effect of b_t. In contrast, the government with discretion under Timing 1 does not consider the disincentive effect because it is considered foregone by the government (as it happens in the previous period).

Proposition 2. The benefit level with discretion under Timing 1 is *higher* than that under Timing 2.

Proof. Similar to the previous proof, the first term in equation (14) is positive, so $\frac{\partial R_t}{\partial b_t} = u_{t+1}(1 - u_{t+1})U^u_{c,t} - (1 - u_{t+1})u_{t+1}U^e_{c,t} > 0$, which is equivalent to $U^u_{c,t} > U^e_{c,t}$. This means the benefit level with discretion under Timing 2 is not high enough to equalize marginal utilities of consumption, as under Timing 1. □

4.3 UI policy with commitment

The Ramsey problem of the government with commitment is similarly defined as under Timing 1. Denote the flow equation (11) by $\tilde{\eta}_0(u_t, s_t, u_{t+1}) = 0$ and the optimal search condition (13) by $\tilde{\eta}_1(u_t, b_t, s_t, s_{t+1}) = 0$. Let $\beta^t\tilde{\lambda}_t$ and $\beta^t\tilde{\mu}_t$ be the Lagrange multipliers on $\tilde{\eta}_0$ and $\tilde{\eta}_1$ respectively. The Ramsey government policy under Timing 2 can be characterized by the flow equation (11), the worker's search condition (13), and the following first-order conditions, for all $t > 0$,

$$b_t: \quad \tilde{\mu}_t \frac{\partial \tilde{\eta}_{1,t}}{\partial b_t} - \frac{\partial R_t}{\partial b_t} = 0 \tag{17}$$

$$s_t: \quad \frac{\tilde{\mu}_{t-1}}{\beta} \frac{\partial \tilde{\eta}_{1,t-1}}{\partial s_t} + \tilde{\lambda}_t \frac{\partial \tilde{\eta}_{0,t}}{\partial s_t} + \tilde{\mu}_t \frac{\partial \tilde{\eta}_{1,t}}{\partial s_t} - \frac{\partial R_t}{\partial s_t} = 0 \tag{18}$$

$$u_{t+1}: \quad \tilde{\lambda}_t \frac{\partial \tilde{\eta}_{0,t}}{\partial u_{t+1}} - \beta\left(\frac{\partial R_{t+1}}{\partial u_{t+1}} - \tilde{\lambda}_{t+1}\frac{\partial \tilde{\eta}_{0,t+1}}{\partial u_{t+1}} - \tilde{\mu}_{t+1}\frac{\partial \tilde{\eta}_{1,t+1}}{\partial u_{t+1}}\right) = 0 \tag{19}$$

Unlike in Timing 1, here the optimal benefit condition for the government with commitment (equation 17) does not have terms with the lagged Lagrange multiplier $\tilde{\mu}_{t-1}$. This is because under Timing 2, the period-t benefit b_t does not have direct effects on the unemployed worker's search in period $t-1$, unlike in Timing 1. This is also why the optimal benefit condition for the government with commitment has the same terms as the benefit condition for the government with discretion (equation 14) under this timing.

However, the lagged multiplier $\tilde{\mu}_{t-1}$ still shows up in the optimal search condition with commitment (equation 18). This is because even though period-t UI benefit does not directly affect period-$t-1$ search, it has an indirect effect: a lower b_t increases period-t search s_t, which in turn increases period $t-1$ search s_{t-1}. The government with commitment internalizes this indirect effect, which is captured by the extra term in (18). The government with discretion does not take into account this indirect effect of b_t, because it treats period $t-1$ as foregone.

Comparing across the two timings, because search takes place before consumption in each period, the main source of time inconsistency is not present under Timing 2. With stronger time inconsistency under Timing 1, the difference between benefit levels with discretion and with commitment is also larger under Timing 1. In a partial equilibrium, we have the following proposition, where superscripts 1 and 2 denote variables under the two timings respectively:

Proposition 3. In a partial equilibrium where the steady state Lagrange multipliers with commitment are equal and the steady state unemployment is also the same under the two timings, i.e., $\tilde{\mu}^1 = \tilde{\mu}^2$ and $u^1 = u^2$, the difference in benefit levels between the policy with discretion and the policy with commitment is *larger* under Timing 1 than under Timing 2 at the steady state.

Proof. Simplifying the optimal benefit conditions for the government with commitment (equations 8 and 17) gives

$$\begin{aligned} \tilde{\mu}^1_{t-1}\left[u^1_t U^{e,1}_{c,t} + (1-u^1_t)U^{u,1}_{c,t}\right] &= u^1_t(1-u^1_t)\left(U^{u,1}_{c,t} - U^{e,1}_{c,t}\right) \\ \tilde{\mu}^2_t\left[u^2_{t+1} U^{e,2}_{c,t} + (1-u^2_{t+1})U^{u,2}_{c,t}\right] &= u^2_{t+1}(1-u^2_{t+1})\left(U^{u,2}_{c,t} - U^{e,2}_{c,t}\right) \end{aligned}$$

Conditional on $\tilde{\mu}^1 = \tilde{\mu}^2$ and $u^1 = u^2$ at the steady state, then the benefit level with commitment is the same between the two timings. We have proven Proposition 2 that the benefit level with discretion under Timing 1 is higher than under Timing 2. Together it means the difference in benefit levels between the policy with discretion and the policy with commitment is larger under Timing 1 than under Timing 2. □

Proposition 3 is conditional on holding the multiplier $\tilde{\mu}$ and the aggregate state u the same for the policy with commitment under both timings; otherwise the proof becomes intractable. Next we verify numerically that the results apply more broadly, where the equilibrium variables can be different under different policies and timings.

5 Numerical Analysis

We assume the utility function is

$$U(c) - v(s) = \frac{c^{1-\sigma}}{1-\sigma} - \alpha\frac{s^{1+\phi}}{1+\phi}.$$

For the baseline parameters, we take $\beta = 0.99$, $\sigma = 1$, $\alpha = 2$, $\phi = 1$, $w = 1$, $h = 0.6$, $\delta = 0.06$ and $f = 1$. These baseline parameter values belong to the ranges of values that are commonly used in the search literature.

Table 1: Baseline – UI Benefits b under Timing 1 versus Timing 2

Policy scenario	Timing 1	Timing 2
(a) with discretion	0.4	0.1911
(b) with commitment	0.1407	0.1408
Difference (a-b)	0.2593	0.0503

Table 1 compares the UI policy with discretion and the UI policy with commitment at the steady state under each timing. First, under both timings, the UI benefit with discretion is higher than the benefit with commitment (Proposition 1). Second, the benefit with discretion is higher under Timing 1 than under Timing 2 (Propositions 2). Third and more importantly, the difference between the policy with discretion and the policy with commitment is substantially larger under Timing 1 than under Timing 2 (Propositions 3), which implies a much stronger time inconsistency effect under Timing 1. Lastly, the benefit with commitment are almost the same under the two timings, which suggests that timing assumption matters little for the policy with commitment, consistent with Chugh (2009)'s finding for the economy with cash-in-advance constraint. Table 2 shows that these findings are robust to changing parameter values.

Table 2: Sensitivity Analysis – UI Benefits b under Timing 1 versus Timing 2

	$\beta = 0.95$		$\beta = 0.999$		$\sigma = 0.5$		$\sigma = 1.5$	
Policy scenario	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2
(a) with discretion	0.4	0.1900	0.4	0.1913	0.4	0.1394	0.4	0.2206
(b) with commitment	0.1368	0.1370	0.1417	0.1417	0.0979	0.0979	0.1677	0.1677
Difference (a-b)	0.2632	0.0530	0.2583	0.0496	0.3021	0.0415	0.2323	0.0529
	$\alpha = 1.5$		$\alpha = 2.5$		$\phi = 0.5$		$\phi = 1.5$	
	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2
(a) with discretion	0.4	0.1808	0.4	0.1988	0.4	0.1669	0.4	0.2021
(b) with commitment	0.1415	0.1416	0.1401	0.1401	0.1161	0.1161	0.1581	0.1581
Difference (a-b)	0.2585	0.0392	0.2599	0.0587	0.2839	0.0508	0.2419	0.0440
	$w = 0.9$		$w = 1.1$		$h = 0.5$		$h = 0.7$	
	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2
(a) with discretion	0.3	0.1364	0.5	0.2470	0.5	0.2531	0.3	0.1324
(b) with commitment	0.0948	0.0949	0.1888	0.1889	0.1986	0.1987	0.0888	0.0889
Difference (a-b)	0.2052	0.0415	0.3112	0.0581	0.3014	0.0544	0.2112	0.0435
	$\delta = 0.04$		$\delta = 0.08$		$f = 0.9$		$f = 1.1$	
	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2	Timing 1	Timing 2
(a) with discretion	0.4	0.1937	0.4	0.1884	0.4	0.1983	0.4	0.1843
(b) with commitment	0.1424	0.1424	0.1392	0.1392	0.1401	0.1401	0.1413	0.1413
Difference (a-b)	0.2576	0.0513	0.2608	0.0492	0.2599	0.0582	0.2587	0.0430

6 Conclusion

This paper contributes to a better understanding of the timing assumption and time inconsistency in search models. We analyze UI policy with discretion and with commitment under two alternative timings. Under the first timing, where consumption takes place before search within each period, time inconsistency arises due to the direct disincentive effect of current-period UI benefit on search in the previous period. Under the second timing, where search takes place before consumption within each period, time inconsistency shows up indirectly, as the current-period benefit affects within-period search, which in turn affects search in the previous period. We show that time inconsistency is stronger under the first timing, and timing assumption matters for the policy with discretion but not much for the policy with commitment. These results have implications for analyzing optimal UI policies with and without commitment.

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